Computer Graphics III – Radiometry

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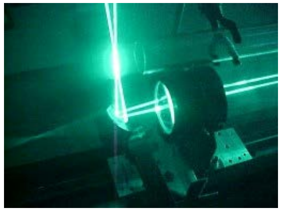
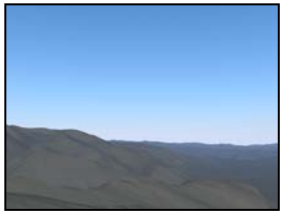
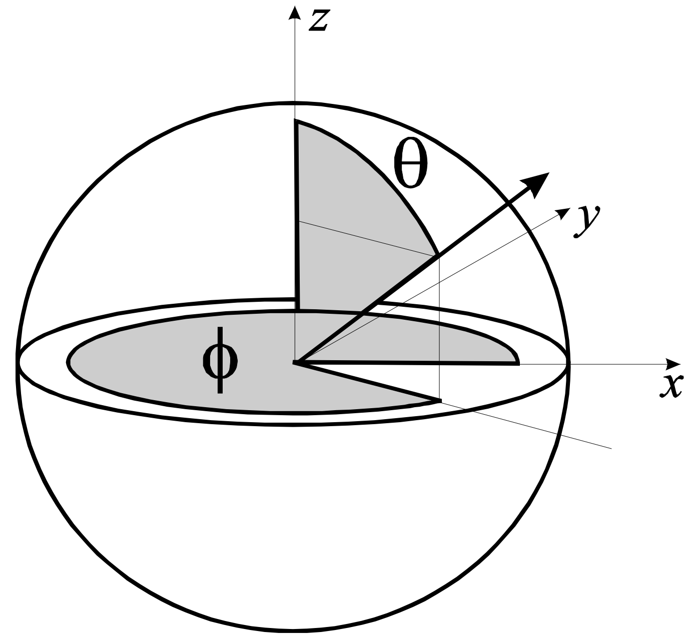
  

Direction, solid angle, spherical integrals

Direction in 3D



- **Direction** = unit vector in 3D
  - Cartesian coordinates

$$\omega = [x, y, z], \quad x^2 + y^2 + z^2 = 1$$

  - Spherical coordinates

$$\omega = [\theta, \varphi]$$

$$\theta \in [0, \pi] \qquad \theta = \arccos z \qquad x = \sin\theta\cos\varphi$$

$$\varphi \in [0, 2\pi] \qquad \varphi = \arctan\frac{y}{x} \qquad y = \sin\theta\sin\varphi$$

$$z = \cos\theta$$

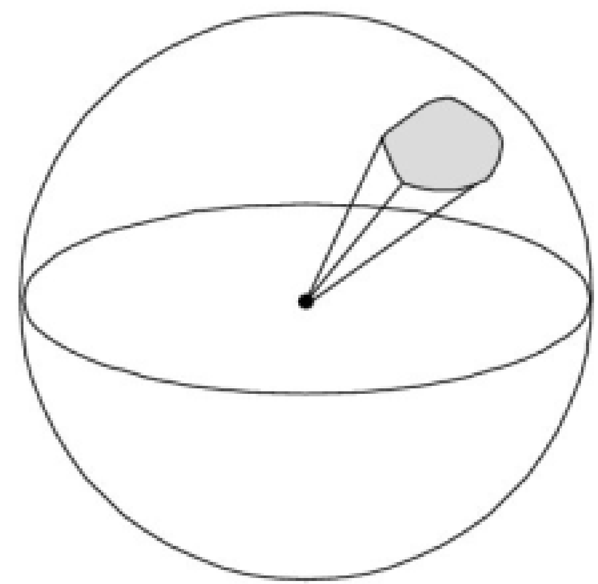
  - θ … *polar angle* – angle from the *Z* axis
  - ϕ … *azimuth* – angle measured counter-clockwise from the *X* axis

Function on a unit sphere

- Function as any other, except that its argument is a direction in 3D
- Notation
  - $F(\omega)$
  - $F(x,y,z)$
  - $F(\theta,\phi)$
  - …
  - Depends in the chosen representation of directions in 3D

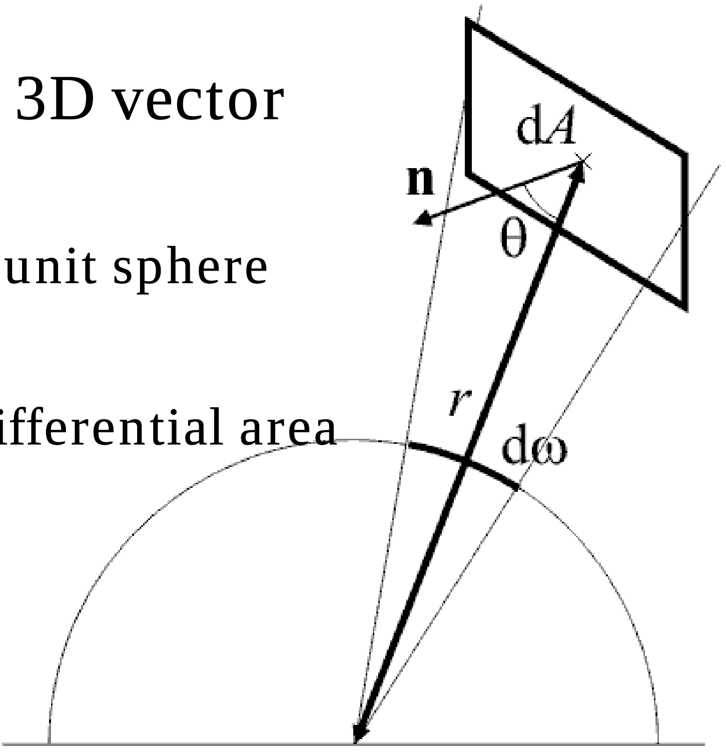
Solid angle

- **Planar angle**
  - Arc length on a unit circle
  - A full circle has 2π radians (unit circle has the length of 2π)
- **Solid angle** (steradian, sr)
  - Surface area on an unit sphere
  - Full sphere has 4π steradians



Differential solid angle

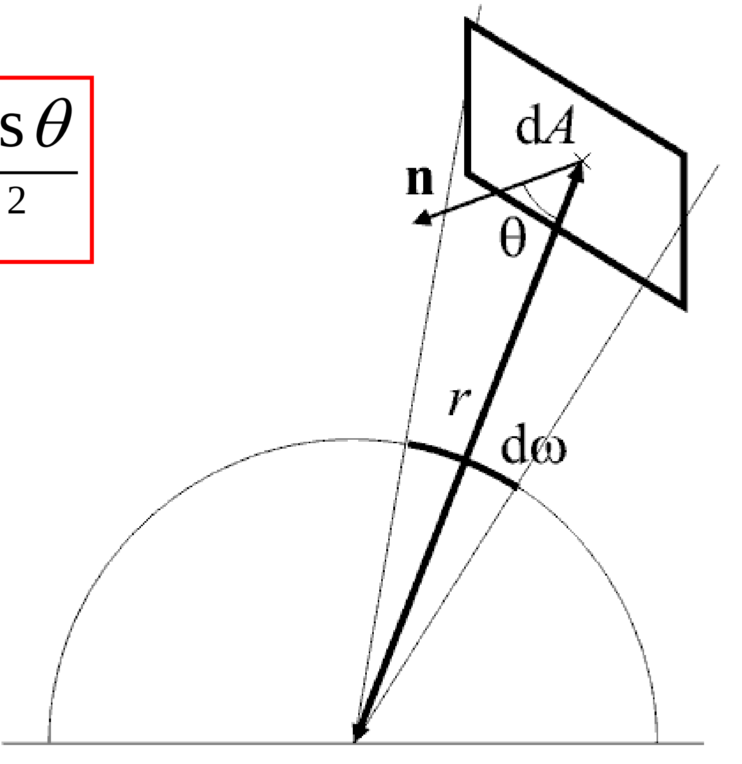
- "Infinitesimally small" solid angle around a given direction
- By convention, represented as a 3D vector
  - Magnitude ... dω
    - Size of a differential area on the unit sphere
  - Direction ... ω
    - Center of the projection of the differential area on the unit sphere



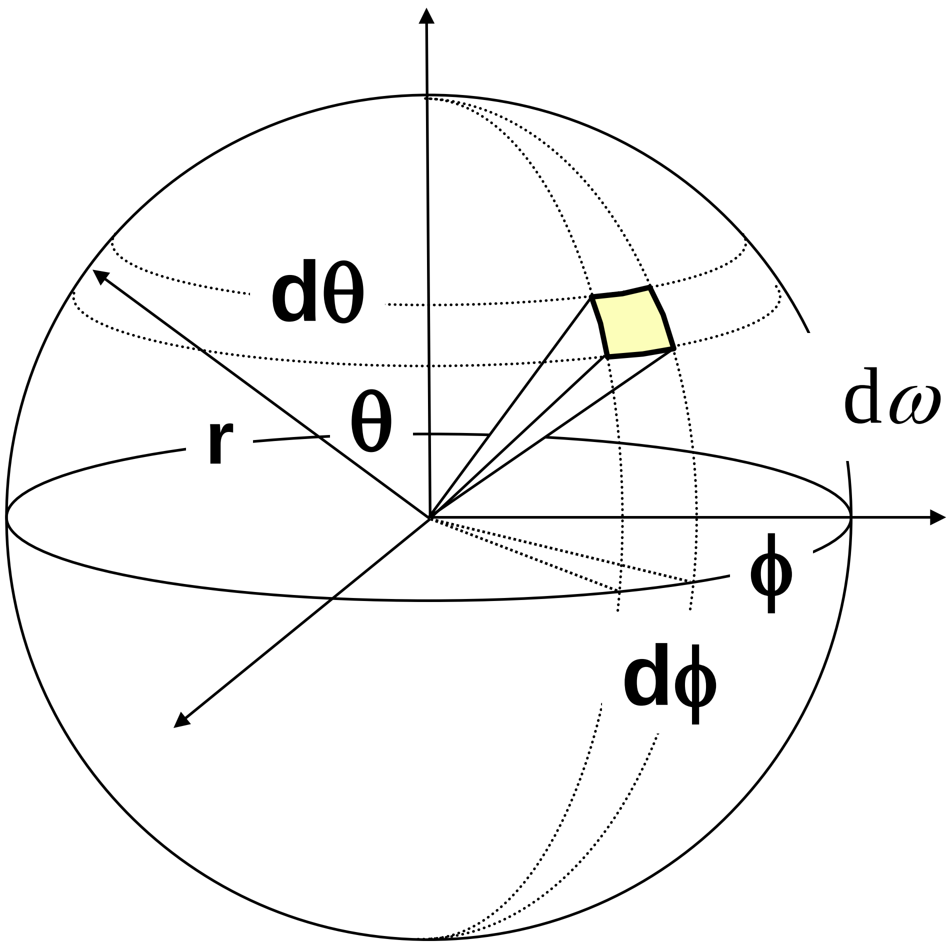
Differential solid angle

- (Differential) solid angle subtended by a differential area

$$\boxed{\mathrm{d}\omega = \mathrm{d}A \frac{\cos\theta}{r^2}}$$



Differential solid angle



$$d\omega = (d\theta)\,(\sin\theta\, d\phi)$$
$$= \sin\theta\, d\theta\, d\phi$$

Radiometry and photometry

Radiometry and photometry

- “**Radiometry** is a set of techniques for measuring electromagnetic radiation, including visible light.
- Radiometric techniques in optics characterize the distribution of the radiation's power in space, as opposed to **photometric** techniques, which characterize the light's interaction with the human eye.”

(Wikipedia)

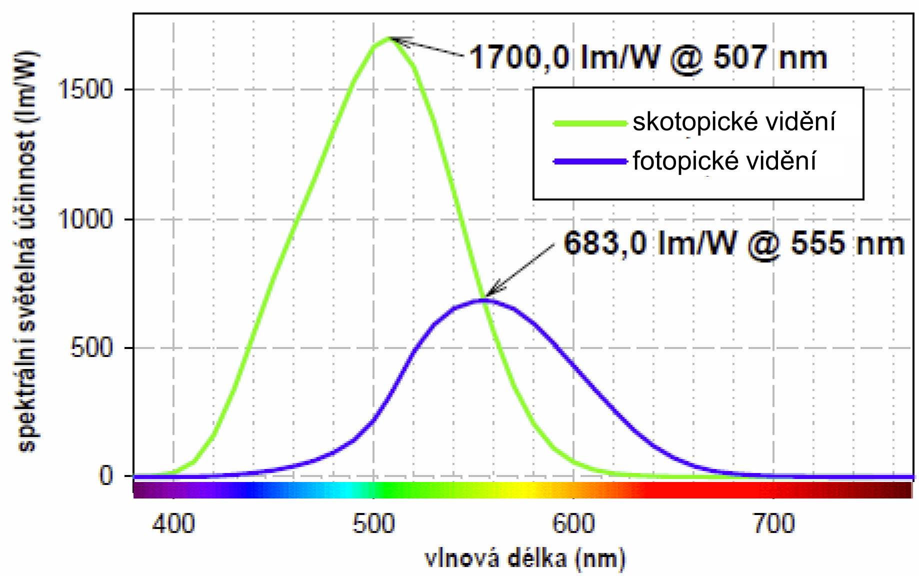
Radiometry and photometry

- **Radiometric quantities**
  - Radiant energy (zářivá energie) – Joule
  - Radiant flux (zářivý tok) – Watt
  - Radiant intensity (zářivost) – Watt/sr
  - *Denoted by subscript **e***
- **Photometric quantities**
  - Luminous energy (světelná energie) – Lumen-second, a.k.a. Talbot
  - Luminous flux (světelný tok) – Lumen
  - Luminous intensity (svítivost) – candela
  - *Denoted by subscript **v***

Relation between photo- and radiometric quantities

- **Spectral luminous efficiency** K(λ)

$$\boxed{K(\lambda) = \frac{d\Phi_{\lambda}}{d\Phi_{e\lambda}}}$$



Obr. 7. Spektrální světelná účinnost při fotopickém (denním) vidění a skotopickém (soumrakovém) vidění.

Source: M. Procházka: Optika pro počítačovou grafiku

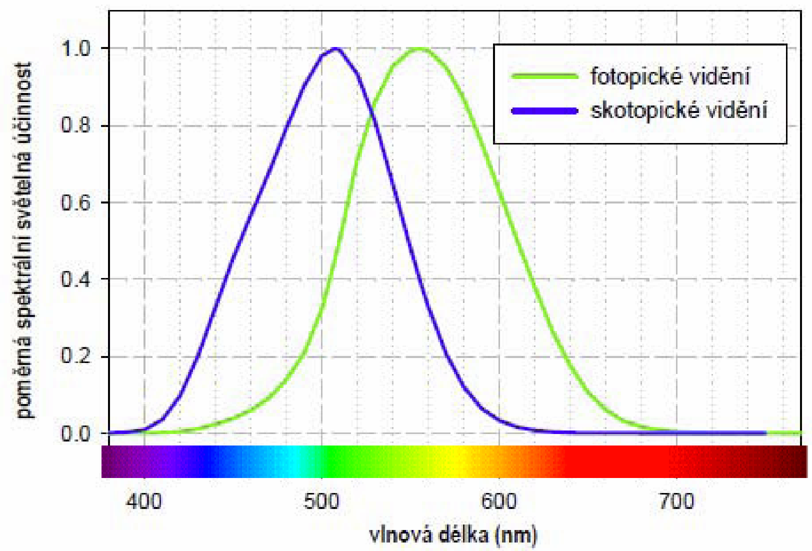
Relation between photo- and radiometric quantities

- Visual response to a spectrum:

$$\boxed{\Phi = \int_{380\,\mathrm{nm}}^{770\,\mathrm{nm}} K(\lambda)\, \Phi_{\mathrm{e}}(\lambda)\, \mathrm{d}\lambda}$$

Relation between photo- and radiometric quantities

- **Relative spectral luminous efficiency** $V(\lambda)$
  - Sensitivity of the eye to light of wavelength λ relative to the peak sensitivity at λ_{max} = 555 nm (for photopic vision).
  - CIE standard 1924



Obr. 6. Poměrná spektrální světelná účinnost při fotopickém (denním) a skotopickém (soumrakovém) vidění.

Source: M. Procházka: Optika pro počítačovou grafiku

Relation between photo- and radiometric quantities

- **Radiometry**
  - More fundamental – photometric quantities can all be derived from the radiometric ones
- **Photometry**
  - Longer history – studied through psychophysical (empirical) studies long before Maxwell equations came into being.

Radiometric quantities

Transport theory

- Empirical theory describing flow of “energy” in space
- **Assumption:**
  - Energy is continuous, infinitesimally divisible
  - Needs to be taken so we can use derivatives to define quantities
- Intuition of the “energy flow”
  - Particles flying through space
  - No mutual interactions (implies linear superposition)
  - Energy density proportional to the density of particles
  - This intuition is abstract, empirical, and has nothing to do with photons and quantum theory

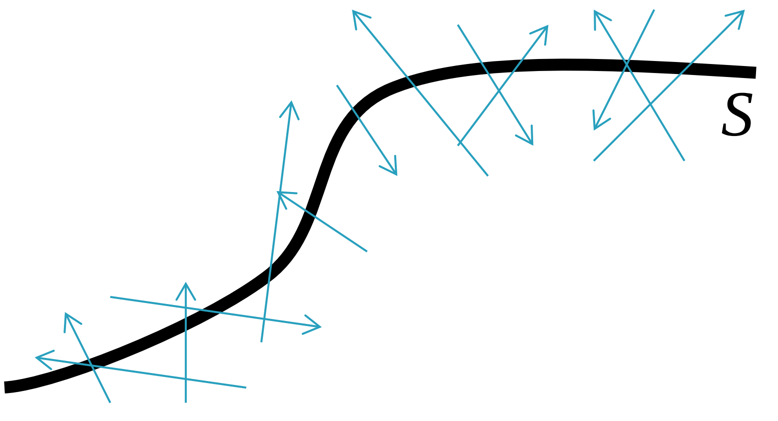
Radiant energy – Q [J]

Time interval

Wavelength interval

$$\boxed{Q\,(S,\ <t_1,\ t_2>,\ <\lambda_1,\ \lambda_2>)}$$

Surface in 3D
(imaginary or real)



- **Unit**: Joule, J

Spectral radiant energy – Q [J]

- Energy of light at a specific wavelength
  - „Density of energy w.r.t wavelength“

$$Q_\lambda(S,\langle t_1,t_2\rangle,\lambda)=\lim_{\substack{d(\lambda_1,\lambda_2)\to 0\\ \lambda\in\langle\lambda_1,\lambda_2\rangle}}\frac{Q(S,\langle t_1,t_2\rangle,\langle\lambda_1,\lambda_2\rangle)}{\mu\langle\lambda_1,\lambda_2\rangle}=\text{formally}=\frac{\mathrm{d}Q}{\mathrm{d}\lambda}$$

- We will leave out the subscript and argument λ for brevity
  - We always consider spectral quantities in image synthesis

- **Photometric quantity**:
  - Luminous energy, unit Lumen-second aka Talbot

Radiant flux (power) – Φ [*W*]

- How quickly does energy „flow“ from/to surface *S*?
  - „Energy density w.r.t. time“

$$\boxed{\Phi(S,t) = \lim_{\substack{d\langle t_1,t_2\rangle \to 0 \\ t\in\langle t_1,t_2\rangle}} \frac{Q(S,\langle t_1,t_2\rangle)}{\mu\langle t_1,t_2\rangle} = (\text{formálně}) = \frac{\mathrm{d}Q}{\mathrm{d}t}}$$

- **Unit**: Watt – *W*
- **Photometric quantity**:
  - Luminous flux, unit Lumen

Irradiance– E [W.m^{-2}]

- What is the spatial flux density at a given point **x** on a surface S?

$$E(\vec{x}) = \lim_{\substack{d(S)\to 0 \\ \vec{x}\in S,\ S\subseteq P}} \frac{\Phi_i(S)}{\mu(S)} = (\text{formálně}) = \frac{\mathrm{d}\Phi_i}{\mathrm{d}S}$$

- Always defined w.r.t some point **x** on S with a specified surface normal N(**x**).
  - **Irradiance DOES depend on *N*(x)** (Lambert law)

- We're only interested in light arriving from the "outside" of the surface (given by the orientation of the normal).

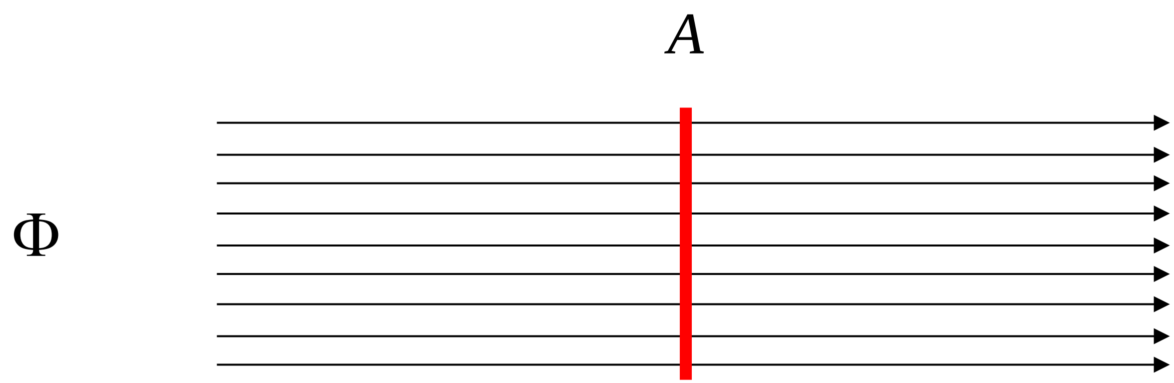
Irradiance – E [W.m^{-2}]

- **Unit**: Watt per meter squared – $W.m^{-2}$
- **Photometric quantity**:
  - Illuminance, unit Lux = lumen.m^{-2}

light meter
(cz: expozimetr)

Lambert cosine law

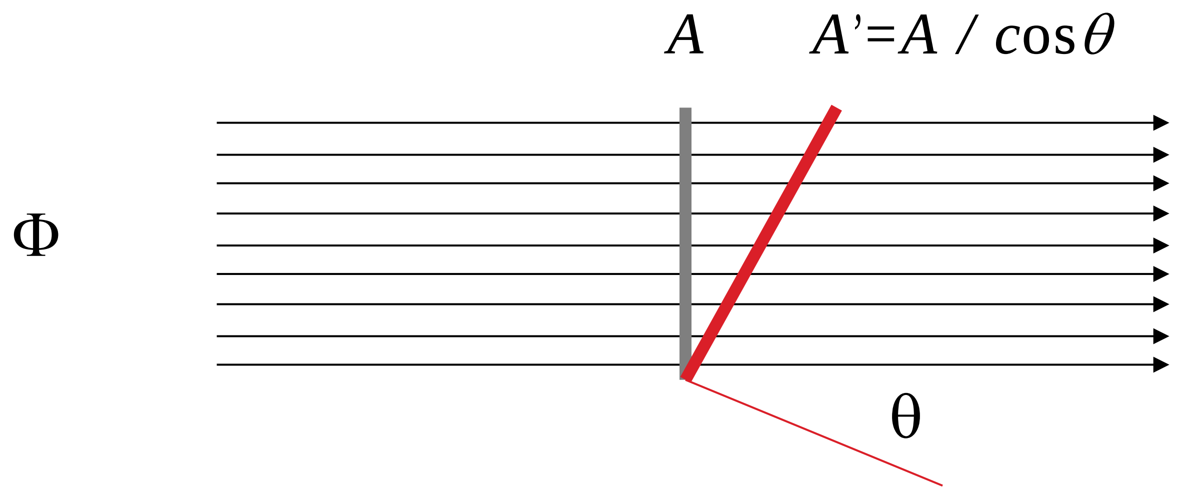
- Johan Heindrich Lambert, *Photometria*, *1760*



$$\boxed{E = \frac{\Phi}{A}}$$

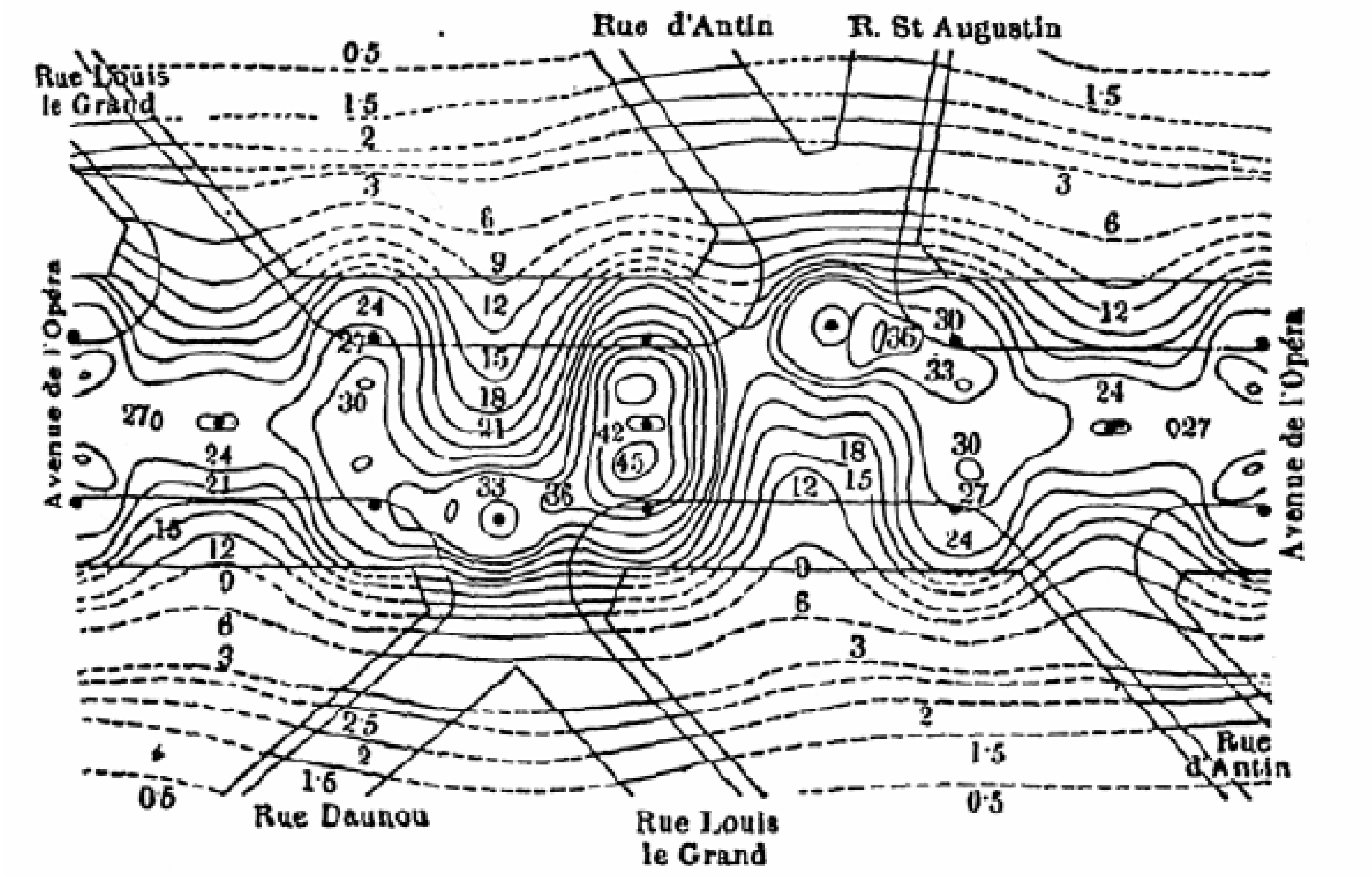
Lambert cosine law

- Johan Heindrich Lambert, *Photometria*, *1760*



$$\boxed{E' = \frac{\Phi}{A'} = \frac{\Phi}{A}\cos\theta}$$

Irradiance Map or Light Map



Isolux contours

Typical Values of Illuminance [lm/m^2]

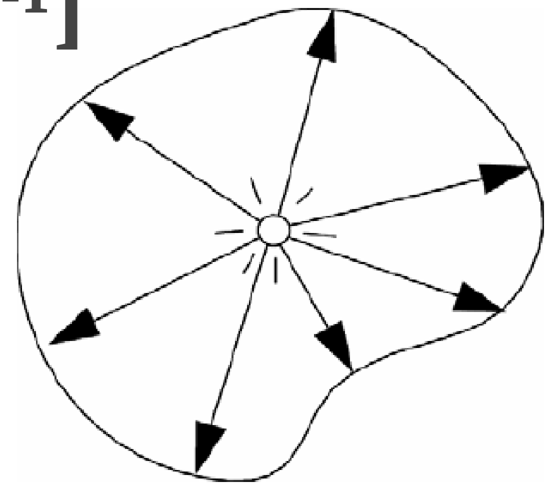
Sunlight plus skylight	100,000 lux
Sunlight plus skylight (overcast)	10,000
Interior near window (daylight)	1,000
Artificial light (minimum)	100
Moonlight (full)	0.02
Starlight	0.0003

Radiant exitance – B [W.m^{-2}]

- Same as irradiance, except that it describes *exitant* radiation.
  - The exitant radiation can either be directly emitted (if the surface is a light source) or reflected.

- **Common name**: **radiosity**
- **Denoted**: B, M
- **Unit**: Watt per meter squared – W.m^{-2}
- **Photometric quantity**:
  - Luminosity, unit Lux = lumen.m^{-2}

Radiant intensity – I [W.sr^{-1}]



- Angular flux density in direction ω

$$\boxed{I(\omega) = \frac{d\Phi(\omega)}{d\omega}}$$

- **Definition:** Radiant intensity is the power per unit solid angle emitted by a point source.

- **Unit**: Watt per steradian – W.sr^{-1}
- **Photometric quantity**
  - Luminous intensity,
    unit Candela (cd = lumen.sr^{-1}), **SI base unit**

Point light sources

- Light emitted from a single point
  - Mathematical idealization, does not exist in nature
- Emission completely described by the radiant intensity as a function of the direction of emission: $I(\omega)$
  - **Isotropic point source**
    - Radiant intensity independent of direction
  - **Spot light**
    - Constant radiant intensity inside a cone, zero elsewhere
  - **General point source**
    - Can be described by a **goniometric diagram**
      - Tabulated expression for $I(\omega)$ as a function of the direction ω
      - Extensively used in illumination engineering

Spot Light

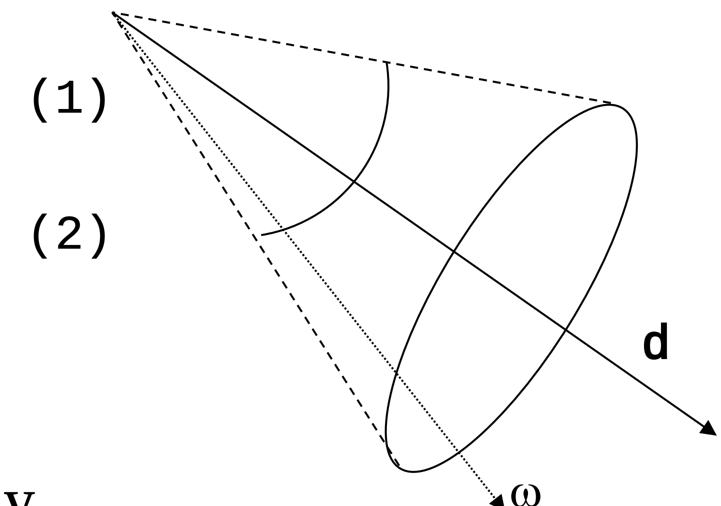
- Point source with a directionally-dependent radiant intensity
- Intensity is a function of the deviation from a reference direction **d** :

$$I(\omega) = f(\angle \omega, \mathbf{d})$$

- E.g.

$$I(\omega) = I_o \cos \angle(\omega, \mathbf{d}) = I_o (\omega \cdot \mathbf{d})$$

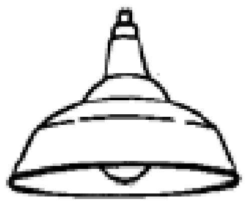
$$I(\omega) = \begin{cases} I_o & \angle(\omega, \mathbf{d}) < \tau \\ 0 & \text{otherwise} \end{cases}$$

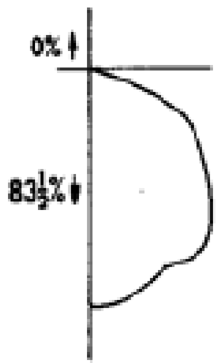


- What is the total flux emitted by the source in the cases (1) a (2)? (See exercises.)

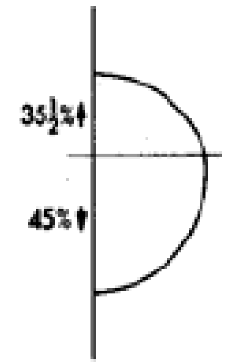
Light Source Goniometric Diagrams

3



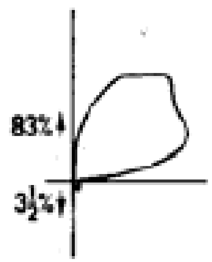


Porcelain-enameled ventilated standard dome with incandescent lamp

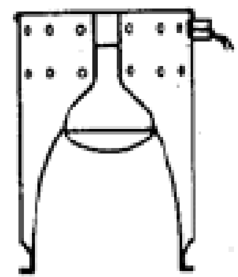


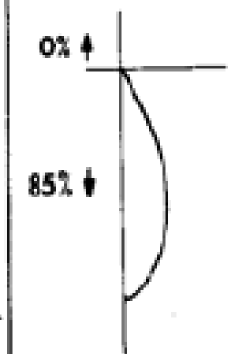
Pendant diffusing sphere with incandescent lamp

2



Concentric ring unit with incandescent silvered-bowl lamp



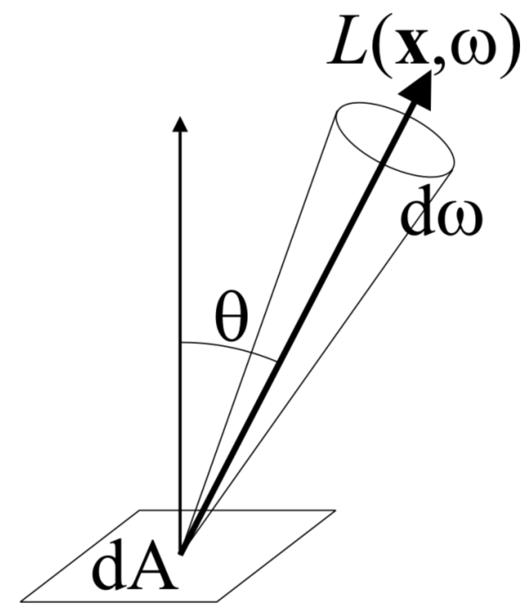


R-40 flood with specular anodized reflector skirt; 45° cutoff

Radiance – L [W.m^{-2}.sr^{-1}]

- Spatial and directional flux density at a given location **x** and direction ω.

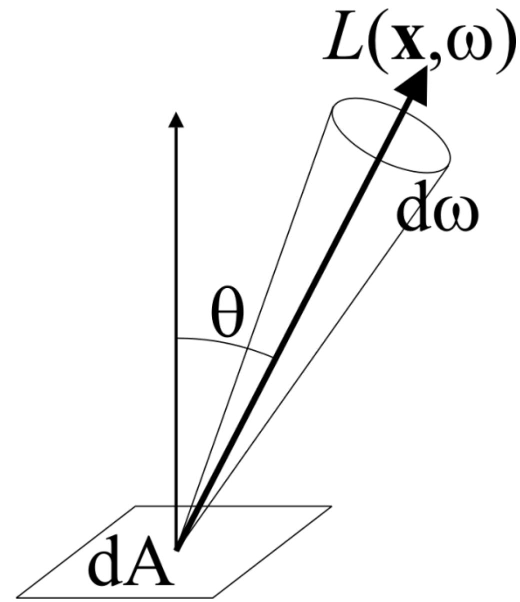
$$\boxed{L(\mathbf{x},\omega)=\frac{d^2\Phi}{\cos\theta\,\mathrm{d}A\,\mathrm{d}\omega}}$$



- **Definition:** *Radiance* is the power per unit area **perpendicular to the ray** and per unit solid angle in the direction of the ray.

Radiance – L [W.m^{-2}.sr^{-1}]

- Spatial and directional flux density at a given location **x** and direction ω.



$$\boxed{L(\mathbf{x},\omega) = \frac{d^2\Phi}{\cos\theta \, \mathrm{d}A \, \mathrm{d}\omega}}$$

- **Unit**: W. m^{-2}.sr^{-1}
- **Photometric quantity**
  - Luminance, unit **candela.m^{-2}** (a.k.a. Nit – used only in English)

The cosine factor cos θ in the definition of radiance

- cos θ compensates for the decrease of irradiance with increasing θ
  - The idea is that **we do not want** radiance to depend on the mutual orientation of the ray and the reference surface

- If you illuminate some surface while rotating it, then:
  - **Irradiance does change with the rotation** (because the actual spatial flux density changes).
  - **Radiance does <u>not</u> change** (because the flux density change is exactly compensated by the cos θ factor in the definition of radiance). And that's what we want.

Typical Values of Luminance [cd/m^2]

Surface of the sun	2,000,000,000 nit
Sunlight clouds	30,000
Clear day	3,000
Overcast day	300
Moon	0.03

The Sky Radiance Distribution

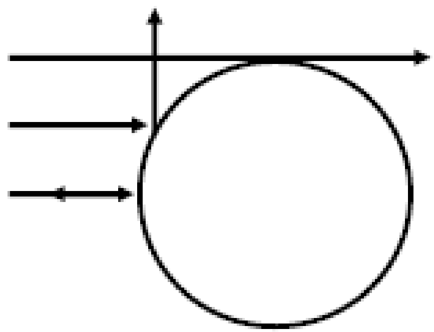
Plate 5-16. Fisheye view of clear sky at the South Pole. (Photographed by the author)

Plate 5-17. View of slightly hazy sky in Wisconsin. (Photographed by the author)

From Greenler, Rainbows, halos and glories

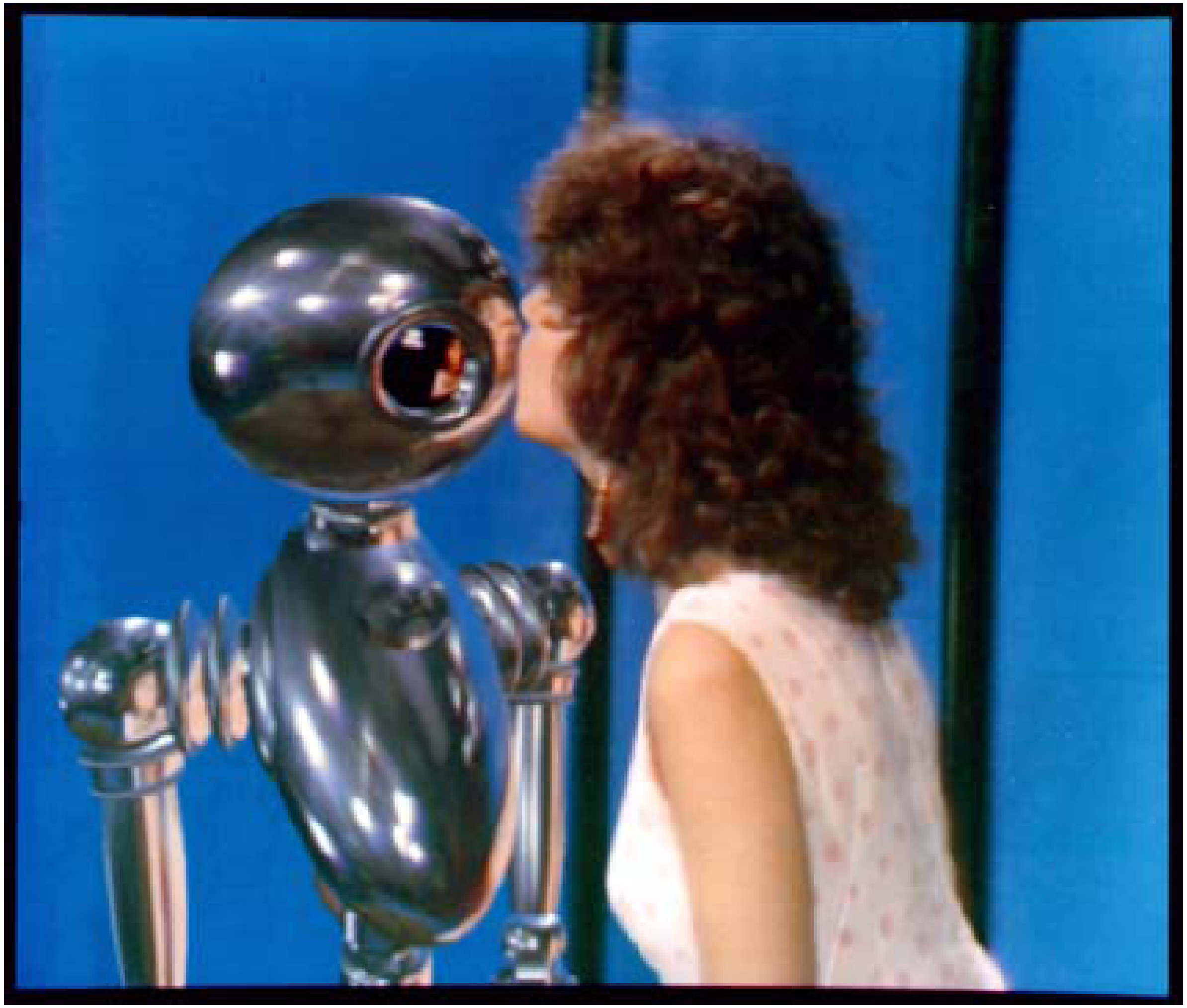
Gazing Ball ⇒ Environment Maps

Miller and Hoffman, 1984



- Photograph of mirror ball
- Maps all spherical directions to a to circle
- Reflection direction indexed by normal
- Resolution function of orientation

Environment Maps



Interface, Chou and Williams (ca. 1985)

Calculation of the remaining quantities from radiance

$$E(\mathbf{x}) = \int_{H(\mathbf{x})} L(\mathbf{x},\omega)\cos\theta \,\mathrm{d}\omega$$

$$\begin{aligned}\Phi &= \int_A E(\mathbf{x})\,\mathrm{d}A_{\mathbf{x}} \\ &= \int_A \int_{H(\mathbf{x})} L(\mathbf{x},\omega)\cos\theta\,\mathrm{d}\omega\,\mathrm{d}A_{\mathbf{x}}\end{aligned}$$

$\cos\theta\,\mathrm{d}\omega$ = projected solid angle

$H(\mathbf{x})$ = hemisphere above the point $\mathbf{x}$

Area light sources

- Emission of an area light source is fully described by the emitted radiance $L_e(\mathbf{x},\omega)$ for all positions on the source $\mathbf{x}$ and all directions ω.

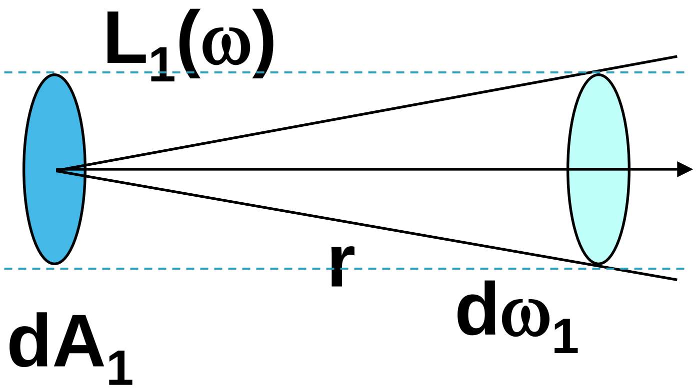
- The total emitted power (flux) is given by an integral of $L_e(\mathbf{x},\omega)$ over the surface of the light source and all directions.

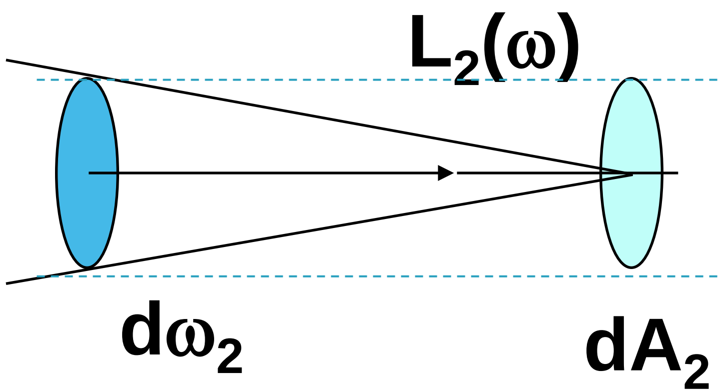
$$\boxed{\Phi = \int_A \int_{H(\mathbf{x})} L_e(\mathbf{x},\omega) \cos\theta \, \mathrm{d}\omega \, \mathrm{d}A}$$

Properties of radiance (1)

- **Radiance is constant along a ray in vacuum**
  - Fundamental property for light transport simulation
  - This is why radiance is the quantity associated with rays in a ray tracer
  - Derived from energy conservation (next two slides)

Energy conservation along a ray



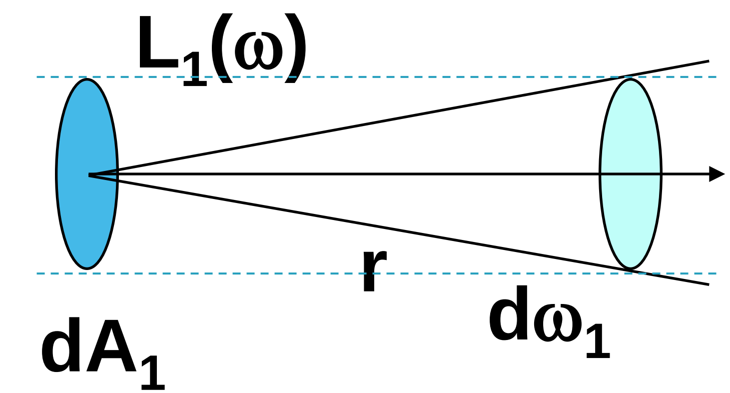


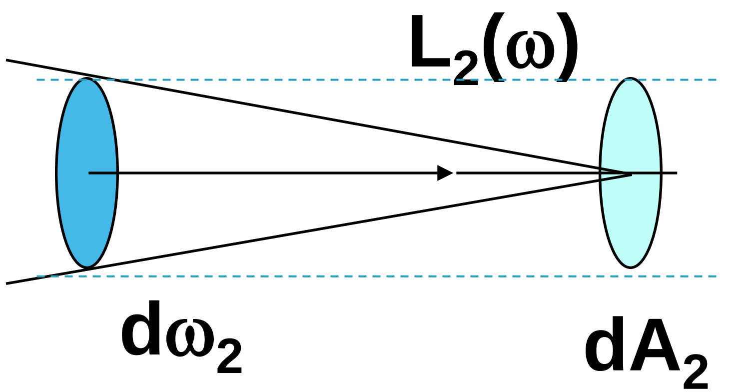
$$L_1\, d\omega_1\, dA_1 = L_2\, d\omega_2\, dA_2$$

emitted flux

received flux

Energy conservation along a ray





$$L_1 \, d\omega_1 \, dA_1 = L_2 \, d\omega_2 \, dA_2$$

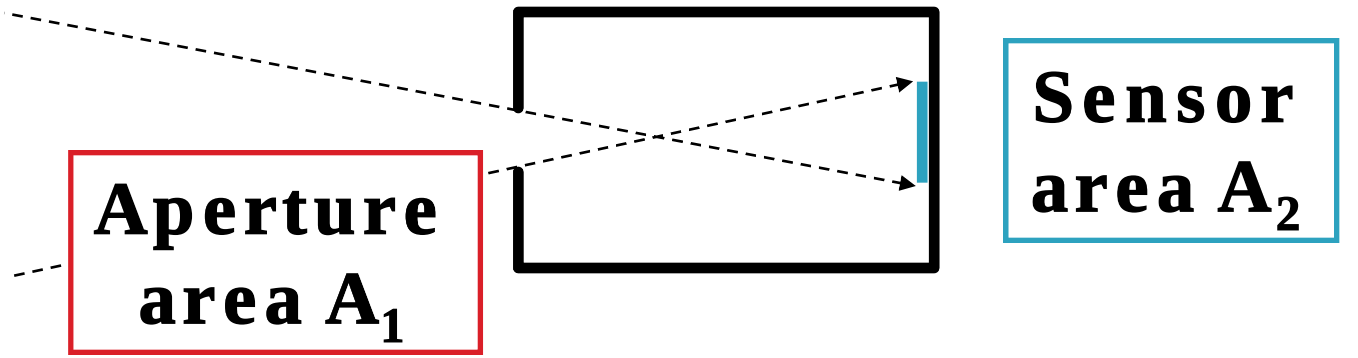
$$T = d\omega_1 \, dA_1 = d\omega_2 \, dA_2 = \frac{dA_1 \, dA_2}{r^2}$$

ray throughput

$$\boxed{L_1 = L_2}$$

Properties of radiance (2)

- **Sensor response** (i.e. camera or human eye) is directly proportional to the value of **radiance** reflected by the surface visible to the sensor.



$$\underline{R} = \int_{A_2} \int_{\Omega} L_{in}(A, \omega) \cdot \cos\theta \, d\omega \, dA = \underline{L_{in} \cdot T}$$

Incoming / outgoing radiance

- Radiance is **discontinuous** at an interface between materials

  - Incoming radiance – $L^i(\mathbf{x},\omega)$
    - radiance just before the interaction (reflection/transmission)

  - Outgoing radiance – $L^o(\mathbf{x},\omega)$
    - radiance just after the interaction

Radiometric and photometric terminology

Fyzika ***Physics***	**Radiometrie** ***Radiometry***	**Fotometrie** ***Photometry***
Energie *Energy*	Zářivá energie *Radiant energy*	Světelná energie *Luminous energy*
Výkon (tok) *Power (flux)*	Zářivý tok *Radiant flux (power)*	Světelný tok (výkon) *Luminous power*
Hustota toku *Flux density*	Ozáření *Irradiance*	Osvětlení *Illuminance*
dtto	Intenzita vyzařování *Radiosity*	??? *Luminosity*
Úhlová hustota toku *Angular flux density*	Zář *Radiance*	Jas *Luminance*
??? Intensity	Zářivost *Radiant Intensity*	Svítivost *Luminous intensity*

Next lecture

- Light reflection on surfaces